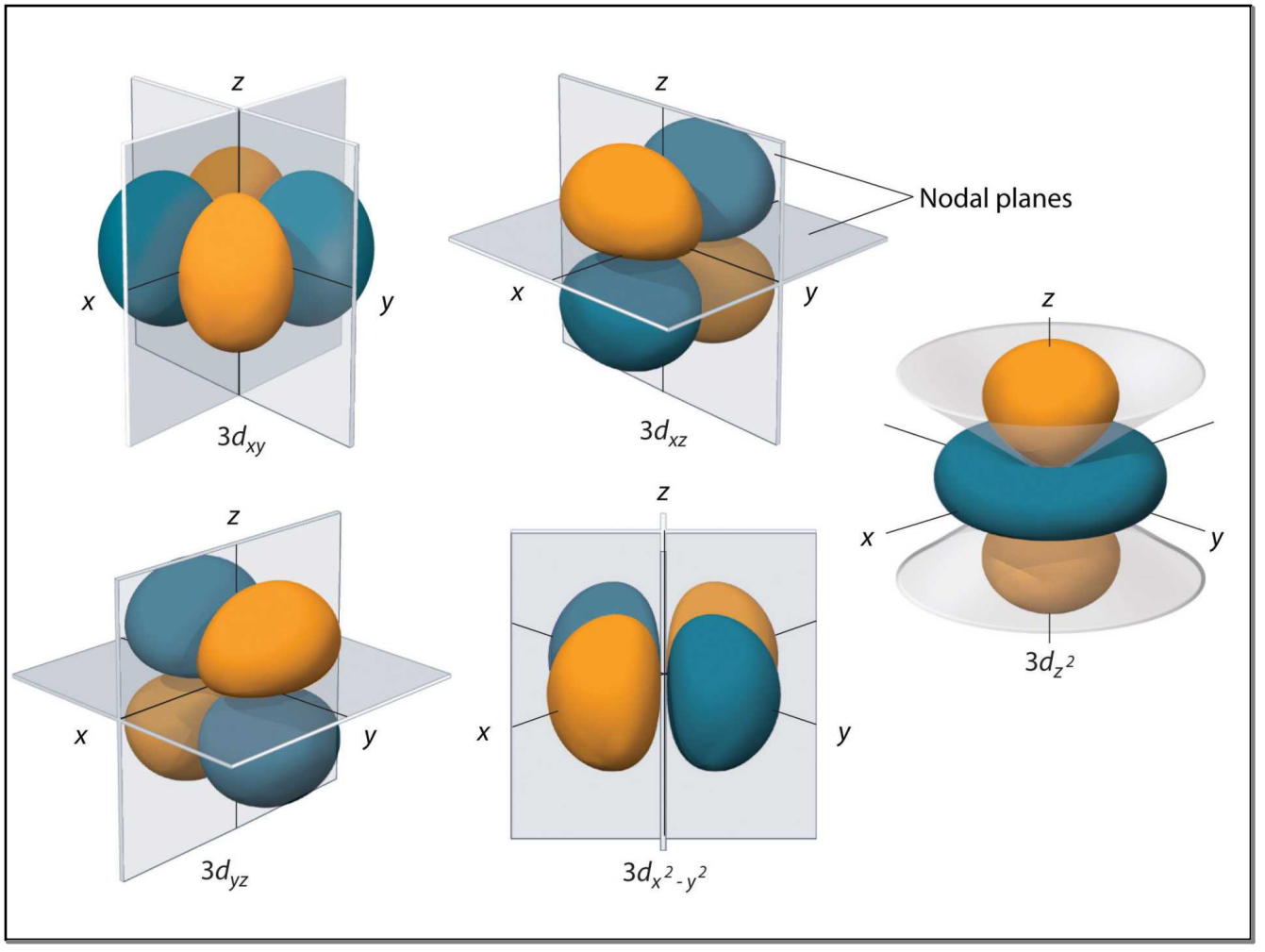

z
x
y
3d_xy
z
x
y
Nodal planes
3d_xz
z
x
y
3d_z²
z
x
y
3d_yz
z
x
y
3d_x²-y²

Probability Distributions

What **we will cover**:

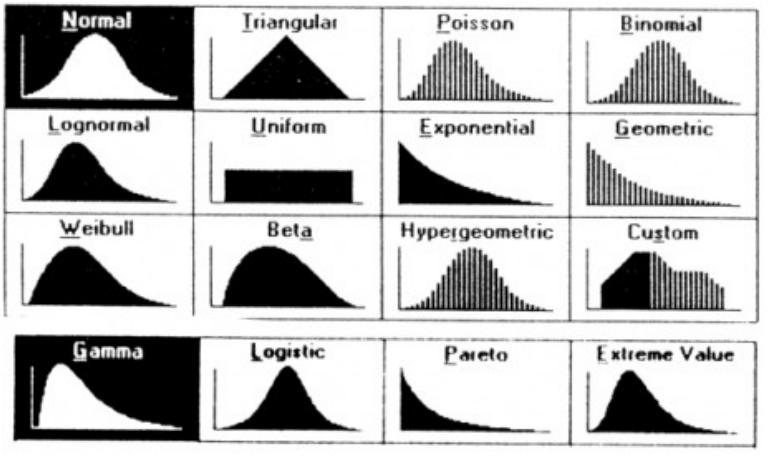
- Types
- Making
- Using
- What they tell you

Intro

A probability distribution is the values a variable can take, along with the probability of each outcome.

There are many types, each with specialized uses.

e.g.
Discrete
Continuous
Binomial
Multinomial
Poisson
Hypergeometric
Normal



Some Uses for Probability Distribution

If you keep track of sales data:

How many people do you need to walk into your store to make your sales quota?

What is the average number of items a person buys?

How likely it is that someone will spend more than a certain dollar amount?

Whats the typical sales amount per sales person?

Vocab

Random variables – a variable whose values are determined by chance.

Discrete – integers or whole numbers

Discrete random variables -

Continuous – real numbers, can take fractional values

Continuous random variables -

Probability Distribution – The values a variable can take, along with the probability of each.

Discrete Probability Distribution

Is the outcome of a coin toss discrete or continuous?

What is the sample space for tossing three coins?

S={ TTT, TTH, THT, HTT, THH, HTH, HHT, HHH }

Let X be the discrete random variable for the number of heads in three tosses, then X can have what values?

X = 0, 1, 2, or 3

What are these probabilities?

P(X = 0) =

P(X = 1) =

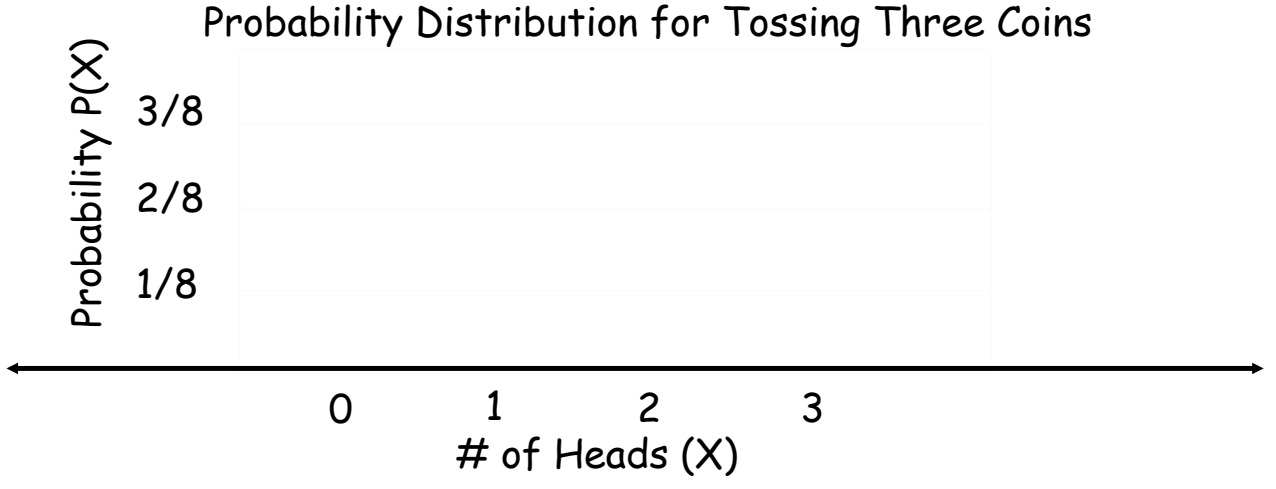
P(X = 2) =

P(X = 3) =

Building the Probability Distribution Table

# of Heads (X)	0	1	2	3
P(X)				

Building the Probability Distribution Graph



You Try

During the summer months (90 days) Bob's Rental rents chainsaws. They keep track of the number of saws rented each day.

Bob's rented 0 saws on 45 days, 1 saw on 30 days, and 2 saws on 15 days.

Let X be the # of saws rented per day.

Construct a Probability Distribution Table and Graph.

Make a probability Distribution Table and Graph for the number of daughters in a family with 4 children.

You can use this info to answer these questions:

What is a more likely family of four – (A) an equal number of boys and girls or (B) 3 of one gender and 1 of the other?

What range of values can P(X) have?

What should all the P(X)s add up to?

It is a requirement that P(X) be between 0 and 1

It is a requirement that the sum of the P(X)s = 1

If either of these fail,
it is <u>not</u> a probability distribution.

Are the following probability distributions?

X	0	5	10	15	20
P(X)	1/5	1/5	1/5	1/5	1/5

X	0	2	4	6
P(X)	-0.3	1.7	0.3	-0.7

X	1	2	3	4
P(X)	¼	1/8	1/16	9/16

X	2	3	7		
P(X)	0.5	0.3	0.4		

Trivia Time

The Real Reasons You Leave College (in order)

1. Too much fun at the expense of classes and grades.
2. A sense of not belonging; a sense of isolation, homesickness.
3. Academically unprepared; burned-out on education.
4. Financial constraints; low on funds.
5. Personal family issues.
6. Academic climate/fit.
7. Choice of wrong major; major not offered.
8. Lack of advising, guidance.
9. Demands from part-time or full-time employment.
10. Move to a different geographic location.

San Diego College of Business Administration

From: www.leavingacademia.com

The United States finished last (46 percent) for the percentage of students who completed college once they started it. That puts the United States behind Japan (89 percent), and former Soviet-bloc states such as Slovakia (63 percent) and Poland (61 percent).

The Harvard study's assertions are supported by data collected by the Organization for Economic Co-operation and Development for its report "Education at a Glance 2010."

Completion Trends 1983–2014

Four-Year Public Colleges—graduation in five years or less

	Highest (%)	Lowest (%)	Current (%)
BA/BS public	52.8 ('86)	36.0 ('13)	36.5
MA/MS public	46.7 ('86)	37.0 ('00)	38.8
PhD public	50.6 ('89, '90)	45.0 ('01)	49.7
BA/BS private	58.5 ('13)	53.3 ('01)	56.7
MA/MS private	58.4 ('88)	53.5 ('01)	54.5
PhD private	68.8 ('86)	62.4 ('14)	62.4

Review time!

Get out a new and freshly scented piece of graph paper, title it:

"Dropping College Courses"

Put your name and period top right.

Copy and complete this table, make a probability distribution graph, then answer the following questions.

Survey question: *What is your primary reason for dropping the class?*
(n = 144)

Reason	Frequency	Relative Frequency
Too Hard	45	
Illness	40	
Work	20	
Change of major	14	
Family	9	
Money	7	
Other	6	
No reason	3	

1. What is the variable being studied?
2. Is the variable random? Explain.
3. How many people were in this study? n =
4. Find these probabilities:
   P(dropping because of illness) =
   P(dropping because of money) =
   P(dropping because of major) =
5. Are the requirements for a discrete probability distribution met? Explain.
6. Are the categories mutually exclusive? Explain.
7. Are the categories exhaustive? Explain.

Moving on...

March 13, 2018

Mean, Standard Deviation and Expected Value

Mean

Remember how to calculate mean?

How do we calculate the mean number of spots showing on a die when it's rolled?

How do we calculate the mean number of spots showing on a die when it's rolled?

In probability distributions the mean is calculated by:

$$\mu = X_1 \bullet P(X_1) + X_2 \bullet P(X_2) + ... + X_n \bullet P(X_n)$$

$$\mu = \sum_{i}^{n} X_i P(X_i)$$

$$\mu = \sum XP(X)$$

You try ...

Find the mean number of spots showing when rolling a fair six-sided die.

$\mu = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$

$\mu = 3.5$

If you win $1 for every spot that showed when you roll the die, how much would you expect to win after 100 rolls?

e.g. Tossing 3 coins at a time, what is the mean number of heads?

S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH)

X, # OF HEADS	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

The mean number of heads is:

$\mu = 0 \bullet 1/8 + 1 \bullet 3/8 + 2 \bullet 3/8 + 3 \bullet 1/8$

$\mu = 0 + 3/8 + 6/8 + 3/8$

μ = 12/8 or 1.5

By keeping track of sales, Slice of Heaven knows the number of toppings a pizza is likely to have. What is the average number of toppings sold per pizza?

30% of pizzas have no toppings
40% have 1 topping
20% have 2 toppings
6% have 3 toppings
4% have 4 or more toppings

= 1.14 toppings

That's it for mean, 

now for standard deviation.

Standard Deviation

To calculate the s.d. of a probability distribution:

$$\sigma = \sqrt{\Sigma(x^2 P(X)) - \mu^2}$$

Using the pizza topping data from the earlier example: 30% have none, 40% have 1, 20% have 2, 6% have 3, and 4% have 4 or more.

Calculate the s.d. of the number of pizza toppings.

σ = 1.04 toppings

You try...

The probability that 0, 1, 2, 3, or 4 people will be queued to go on air when they call a radio talk show is shown in the probability distribution. Find the mean and s.d. for the data. The radio station has four phone lines. When all the lines are full, the caller cannot get through.

X	0	1	2	3	4
P(X)	0.18	0.34	0.23	0. 21	0.04

1.6 and 1.1

Answer

Mean $\mu = \Sigma X \cdot P(X)$

$= 0 \cdot 0.18 + 1 \cdot 0.34 + 2 \cdot 0.23 + 3 \cdot 0.21 + 4 \cdot 0.04$

$= 1.6$

Variance $\sigma^2 = \Sigma X^2 \cdot P(X) - \mu^2$

$= 0^2 \cdot 0.18 + 1^2 \cdot 0.34 + 2^2 \cdot 0.23 + 3^2 \cdot 0.21 + 4^2 \cdot 0.04 - 1.6^2$

$= 3.79 - 2.56$

$= 1.2$

S.D. $\sigma = 1.1$

Given this data, should the station consider getting more phone lines ?

Given this data, should the station consider getting more phone lines ?

No.

The mean number of people on hold is 1.6. Considering the s.d. most callers would be accommodated by the 4 lines because $\mu \pm 2\sigma$ would be 1.6 ± 2.2 That's 0 to 3.8, and that's less than 4. So 95% will get through or be put on hold.

The Demens Vacca Bistro wants to know if it has the right number of waiters. They did a survey of the number of occupied tables (X) at 15-minute intervals and processed it to come up with this probability distribution.

X	0	3	5	6	9	10	12
P(X)	0.02	0.08	0.27	0.32	0.25	0.01	0.05

How many waiters should they have to achieve about 90% coverage? A waiter can serve up to 3 tables at a time.

That's it for mean and s.d. on to

Expected Value

Expected Value

The expected value of a random variable is the theoretical average of the variable.

If that sounds like “mean” you're right, it's the same thing.

Expected value $E(x) = \mu = \Sigma(X \cdot P(X))$

e.g. 1
One thousand raffle tickets are sold at $1 each for a TV valued at $350. What is the expected value of the winnings if a person purchases one ticket?

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You can answer this question by building a prob. dist table.

	win	lose
X, Gain	$349	-$1
P(X)	1/1000	999/1000

Then calculate the expected value.

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X, Gain	$349	-$1
P(X)	1/1000	999/1000

Then calculate the expected value.

E(X) = $349 • 1/1000 + (-$1)•999/1000
= -$0.65

Note: the $349 win is less the cost of the $1 ticket.

This doesn't mean you will lose 65c if you buy a ticket, but that if you bought a ticket regularly the average loss will be 65c per ticket. You can only win, on average, $350 once every 1000 tickets, you lose $1 999 times.

e.g. 2

A ski resort loses $70,000 per season in low-snow years, and makes $250,000 profit on normal or good-snow years. The probability of snowing enough for a good season is 40%. Find the expected profit.

answer

X, Profit	$250,000	-$70,000
P(X)	0.40	0.60

$$E(X) = 250{,}000 \cdot 0.40 + (-70{,}000) \cdot 0.60 = \$58{,}000$$

e.g. 3

One thousand tickets are sold at $2 each for four prizes: $100, $50, $25, and $10.

What is the expected winning amount?

X, gain	$98	$48	$23	$8	-$2
P(X)	0.001	0.001	0.001	0.001	0.996

$$E(X) = (98 \cdot 0.001) + (48 \cdot 0.001) + (23 \cdot 0.001) + (8 \cdot 0.001) + (-2 \cdot 0.996)$$

$$= -\$\ 1.815$$

e.g. 4 Keno

Keno is a favorite casino game. Balls numbered 1 to 80 are tumbled in a machine and 20 are selected at random. For $1 you can play “Mark 1 Number”, where you choose one number and if it is the one of the 20 selected, you win $3.

What's the casino's expected “take” on this game?

Gain, X	-$2	$1
P(X)	.25	.75
X*P(X)	-.50	.75

E(X) = -.50 + .75 = .25
So the casino's take is 25c per game.

Here are some expected values for casino games.

Keno: the house wins about 27c per dollar bet.
Craps: the house wins about 88c per dollar bet.
Roulette: the house wins about 90c per dollar bet.
Chuck-a-Luck: the house wins about 52c per dollar bet.

2014 Actuary Tables (US SSA)

Age	P(death, males)	P(death, females)
15	0.000441	0.000207
16	0.000562	0.000245
17	0.000690	0.000282
18	0.000820	0.000318
19	0.000949	0.000352

http://www.ssa.gov/oact/STATS/table4c6.html

What should you pay for a one year $250,000 life insurance policy?

Your turn.

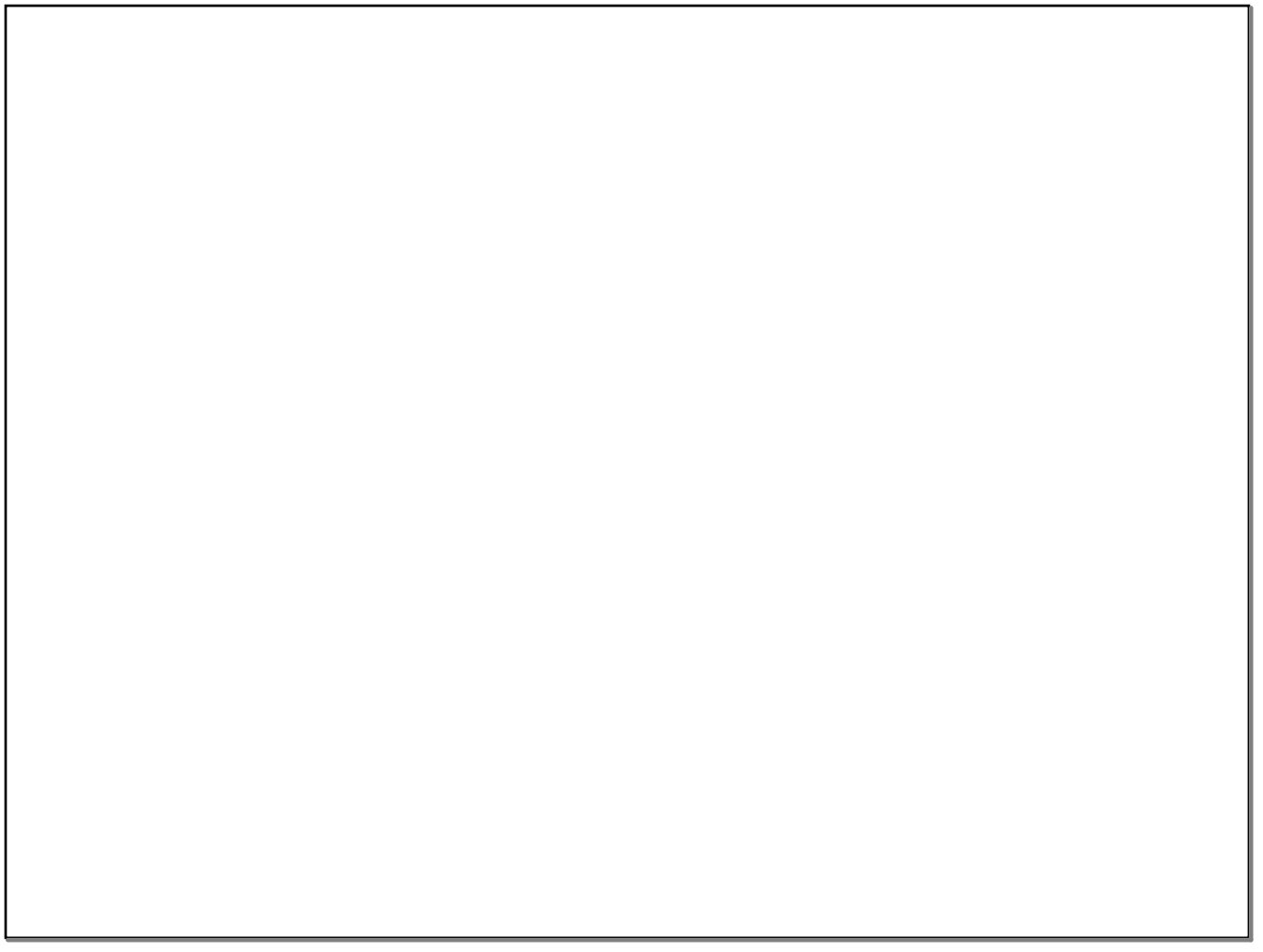
Pick-4

No naked numbers!

Turn in, with cover sheet attached.

That's all folks!

March 13, 2018



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